

Machian strings as an alternative to dark energy

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The expansion history of the Universe is calculated using a simple model in which the entire rest mass energy of a massive particle is distributed throughout the set of Machian strings connecting it to all the other particles in the observable Universe. With the assumption that the energy in a Machian string has the form of positive Newtonian potential energy, the deceleration rates in the radiation and matter eras are exactly the same as in the conventional Λ CDM model. The transition from deceleration to acceleration at the present time is obtained by making the simplest possible modification of the Newtonian potential energy to represent the effect of the cosmological expansion. The effective dark matter and dark energy densities are calculated in terms of the speed of the Hubble flow at the radius of the observable Universe.

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I. INTRODUCTION

It has been known for long time [1] that the Newtonian constant of gravitation, G , satisfies a Machian relation of the form

$$\frac{GM_U}{R_U c^2} \sim 1, \quad (1)$$

where M_U and R_U are, respectively, the mass and radius of the observable Universe at the present time. The purpose of the present paper is to point out that if the relation (1) is assumed to hold at all times then it can be used to calculate the expansion history of the Universe.

In Section II, the relation (1) is used to motivate a new model for an elementary particle. The calculation of the scale factor is described in Section III and it is shown that the entire expansion history of the Universe in the conventional Λ CDM model can be reproduced almost exactly.

II. THE MACHIAN STRING MODEL

If the relation (1) is rewritten as

$$\frac{GmM_U}{R_U} \sim mc^2 \quad (2)$$

then the right hand side is clearly the rest mass energy of a particle of rest mass m and the left hand side may be interpreted as the magnitude of the total Newtonian gravitational interaction energy of the mass m with the observable Universe. The relation (2) may be given a direct physical interpretation in terms of a new model for an elementary particle. Suppose that, instead of being point-like, an elementary particle actually consists of a point-like charged centre together with a set of Machian strings joining the centre to the centres of all other particles in the observable Universe. The entire rest mass energy of the particle is assumed to be distributed within

the strings. If the strings are assumed to contain positive Newtonian potential energy, the condition that all the mass is in the strings for a particle of mass m_i is

$$\sum_j \frac{Gm_i m_j}{r_{ij}} = m_i c^2, \quad (3)$$

where r_{ij} is the distance between the i^{th} and j^{th} particles and the sum is over all particles in the observable Universe. If ρ is the average mass density in the Universe then

$$M_U = \frac{4\pi}{3} \rho R_U^3 \quad (4)$$

and the continuum approximation gives

$$\sum_j \frac{m_j}{r_{ij}} \approx \int_0^{R_U} \frac{\rho}{r} dV = \frac{3M_U}{2R_U}. \quad (5)$$

Equation (3) then becomes, for a particle of mass m ,

$$\frac{3GmM_U}{2R_U} = mc^2. \quad (6)$$

As shown below, equation (6) implies a decelerating Universe. A transition from deceleration at early times to an accelerating expansion at the present time can be obtained by including a dependence of the string energies on the rate of cosmological expansion. The simplest possible modification of the Newtonian potential energy in the string joining masses m_i and mass m_j has the form

$$\frac{Gm_i m_j}{r_{ij}} \left(1 + a \frac{H r_{ij}}{c} \right), \quad (7)$$

where H is the Hubble parameter and a is a free parameter to be determined. The condition that all the mass is in the strings becomes

$$\sum_j \frac{Gm_i m_j}{r_{ij}} \left(1 + a \frac{H r_{ij}}{c} \right) = m_i c^2 \quad (8)$$

which gives the equation

$$\frac{3GmM_U}{2R_U} \left[1 + \frac{2a}{3} \left(\frac{H R_U}{c} \right) \right] = mc^2. \quad (9)$$

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III. CALCULATION OF THE SCALE FACTOR

The radius R_U of the observable Universe is defined by the usual formula

$$R_U(t) = R(t) \int_0^t \frac{c ds}{R(s)}, \quad (10)$$

where $R(t)$ is the scale factor describing the expansion. The total mass M_U of the observable Universe is

$$M_U = \frac{4\pi}{3}(\rho_b + \rho_r)R_U^3, \quad (11)$$

where ρ_b is the baryon density and ρ_r is the effective mass density of radiation. Substituting (11) into equation (9), the equation for $R(t)$ becomes

$$2\pi G(\rho_b + \rho_r)R_U^2 \left[1 + \frac{2a}{3} \left(\frac{HR_U}{c} \right) \right] = c^2. \quad (12)$$

The parameters Ω_b and Ω_r may be defined in the same way as in the standard Λ CDM model, namely

$$\Omega_b = \frac{8\pi G \rho_b^0}{3H_0^2} \quad \text{and} \quad \Omega_r = \frac{8\pi G \rho_r^0}{3H_0^2}, \quad (13)$$

where ρ_b^0 and ρ_r^0 are the densities at the present time and H_0 is the present value of the Hubble parameter. If the scale factor is chosen so that $R = 1$ at the present time then $\rho_b = \rho_b^0/R^3$ and $\rho_r = \rho_r^0/R^4$. Equation (12) then takes the form

$$\frac{3}{4} \left(\frac{H_0 R_U}{c} \right)^2 \left(\frac{\Omega_b}{R^3} + \frac{\Omega_r}{R^4} \right) \left[1 + \frac{2a}{3} \left(\frac{HR_U}{c} \right) \right] = 1. \quad (14)$$

The scale factor was calculated from equation (14) for a range of values of a as described in Appendix A using $H_0 = 70 \text{ km/s/Mpc}$ and $\Omega_b = 0.047$ [2] and $\Omega_r = 8.6 \times 10^{-5}$ [3]. For $a = 0.5$, the scale factor is almost identical to the scale factor in the Λ CDM model, as shown in Figure 1.

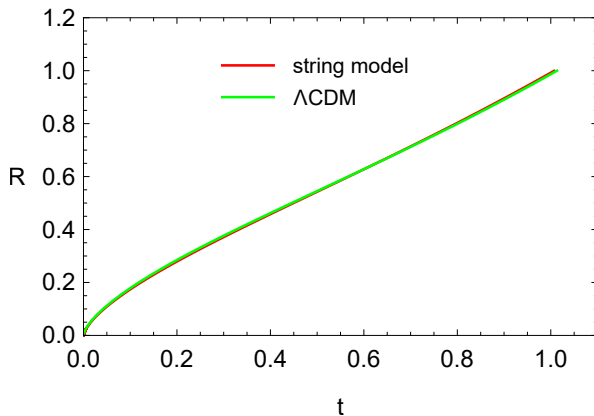


FIG. 1: The scale factor $R(t)$ in the string model with $a = 0.5$ and in the Λ CDM model. The present time corresponds to $R = 1$ and the two scale factors are almost identical.

A plot of residual magnitude against redshift for the two models is shown in Figure 2 together with the experimental data. The acceleration parameter at the present time, $q = R\ddot{R}/\dot{R}^2$, is equal to 0.54 in the string model with $a = 0.5$ and 0.62 in the Λ CDM model.

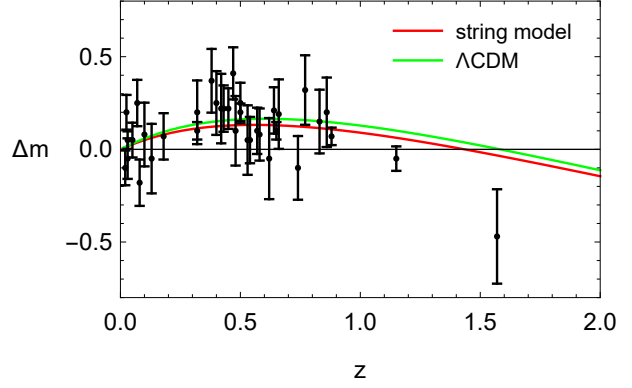


FIG. 2: Plot of residual magnitude against redshift for the string model with $a = 0.5$ and the Λ CDM model, together with the experimental data points and error bars from the Supernova Cosmology Project [4] and the High- z Supernova Search Team [5].

The agreement between the string model and the Λ CDM model is even closer than suggested by Figure 1 and Figure 2. In addition to reproducing the acceleration of the Universe at the present time, Figure 3 shows that the string model also reproduces the $R \sim t^{2/3}$ time evolution of the scale factor in the matter era and the $R \sim t^{1/2}$ time evolution of the scale factor in the radiation era.

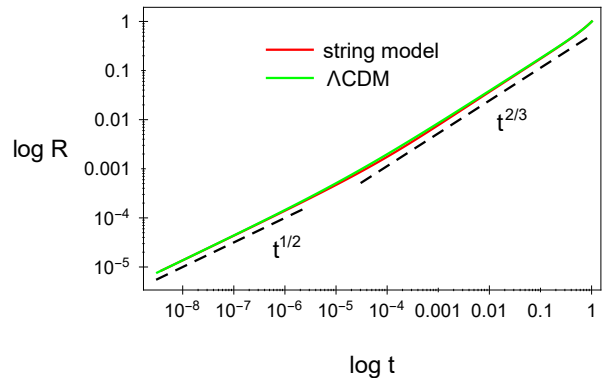


FIG. 3: Plot of $\log R$ against $\log t$ for the best-fit scale factors $R(t)$ in the string model and in the Λ CDM model. In both models, the scale factor is proportional to $t^{1/2}$ in the radiation era and $t^{2/3}$ in the matter era.

It is remarkable that the simple equation (9), with the single parameter a , is able to reproduce exactly the same time evolution of the scale factor in the radiation and matter eras as in the Λ CDM model and also the transition from deceleration to acceleration at late times.

IV. DISCUSSION

A. The time evolution of the scale factor

The power law solutions shown in Figure 3 may be derived from equation (14) as follows. Consider a decelerating solution of the form $R(t) \propto t^\alpha$, for some constant $\alpha < 1$. Then $H = \alpha/t$ and equation (10) gives $R_U \propto t$, from which it follows that the ratio HR_U/c is constant. In the radiation era, the term proportional to Ω_b in equation (14) is negligible and, in the matter era, the term proportional to Ω_r in (14) is negligible. It follows that, to a good approximation, R_U^2/R^4 is constant in the radiation era and R_U^2/R^3 is constant in the matter era, giving $R \propto t^{1/2}$ and $R \propto t^{2/3}$, respectively.

The accelerating solution may be identified by noting that, when $R(t)$ increases faster than t , the integral in (10) tends to a constant at late times. The ratio R_U/R therefore tends to a constant and it follows from equation (14), since the term proportional to Ω_r is negligible, that H tends to a constant. The string model therefore has an accelerating solution at late times of the form $R \sim e^{Ht}$.

B. Comparison with the Friedmann equation

In the conventional Λ CDM model, the scale factor is determined by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_r + \rho_d + \rho_\Lambda), \quad (15)$$

where ρ_d and ρ_Λ are the contributions to the total mass density from ‘dark matter’ and ‘dark energy’, respectively. The Friedmann equation may be compared with equation (12), which may be written in the form

$$H^2 = \frac{8\pi G}{3}(\rho_b + \rho_r) \frac{3}{4} \left(\frac{HR_U}{c} \right)^2 \left[1 + \frac{2a}{3} \left(\frac{HR_U}{c} \right) \right]. \quad (16)$$

Equation (16) may be viewed as a modified Friedmann equation in which the dark matter and dark energy contributions are replaced by a dependence on the quantity HR_U/c , namely the speed of the cosmological expansion at the radius of the observable Universe divided by the speed of light.

In the radiation era, ρ_r is much larger than all the other density components and $R(t) \propto t^{1/2}$ gives $R_U(t) = 2ct$ and $H = 1/2t$ so $HR_U/c = 1$. Equation (16) then gives

$$H^2 \approx \frac{8\pi G}{3} \left(\frac{3+2a}{4} \right) \rho_r, \quad (17)$$

which is equivalent to the Friedmann equation with a rescaled value of ρ_r . The Friedmann equation is recovered when $a = 0.5$, which explains the value of a identified in Section III.

In the matter era, $R(t) \propto t^{2/3}$ gives $R_U(t) \approx 3ct$ and $H = 2/3t$ so $HR_U/c \approx 2$. Equation (16) then reduces to

$$H^2 \approx \frac{8\pi G}{3} (3+4a) \rho_b \quad (18)$$

which is equivalent to the Friedmann equation with a dark matter density

$$\rho_d = (2+4a)\rho_b. \quad (19)$$

In the accelerating era, the factor HR_U/c becomes very large and ρ_b is much larger than ρ_r so equation (16) is equivalent to the Friedmann equation with a dark energy density

$$\rho_\Lambda = \frac{a}{2} \left(\frac{HR_U}{c} \right)^3 \rho_b. \quad (20)$$

As noted above, the quantities H and R_U/R are constant in the accelerating era. Since $\rho_b \propto 1/R^3$, the energy density (20) is constant as the Universe expands.

V. CONCLUSION

The time evolution of the scale factor in the conventional Λ CDM model can be reproduced very accurately if the entire mass-energy of every massive particle is assumed to be distributed within the Machian strings connecting it to all the other particles in the observable Universe. The deceleration in the radiation and matter eras and the transition to acceleration at the present time may all be obtained without ‘dark matter’ or ‘dark energy’. Instead, there is a single parameter representing the effect of the cosmological expansion on the Newtonian potential energy in the strings.

VI. ACKNOWLEDGEMENTS

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Appendix A: Calculation of the scale factor

1. Evolution equation

It is convenient to define the dimensionless time, τ , by $\tau = H_0 t$ and the dimensionless conformal time, η , by the equation

$$\eta(t) = H_0 \int_0^t \frac{ds}{R(s)}. \quad (A1)$$

Then $H_0 R_U/c = R\eta$ and $HR_U/c = \dot{R}\eta$, where the dot denotes differentiation with respect to τ . Substituting into (14) and solving for \dot{R} gives

$$\dot{R}\eta = \frac{2R^2}{a\Omega_b\eta^2(R+R_{eq})} - \frac{3}{2a}, \quad (A2)$$

where $R_{eq} = \Omega_r/\Omega_b$ is the value of R at which the baryon and radiation densities are equal.

2. Numerical integration

To integrate (A2) numerically over several orders of magnitude in time it is convenient to change the independent variable from τ to $u = \ln \eta$, which gives

$$\frac{dR}{du} = \frac{2R^3}{a\Omega_b\eta^2(R + R_{eq})} - \frac{3R}{2a}. \quad (\text{A3})$$

The integration begins at the present time, $R = 1$, and continues back to the singularity at $R = 0$. The value of η at the present time, $\eta = \eta_0$, is chosen so that the Hubble parameter is equal to the observed H_0 , i.e. so that $\dot{R}/R = 1$. From (A2), the required value of η_0 satisfies the equation

$$\eta_0^2 \left(1 + \frac{2a}{3}\eta_0\right) = \frac{4}{3\Omega_b(1 + R_{eq})}, \quad (\text{A4})$$

which has a unique positive solution for η_0 for any non-negative value of a .

The integration was performed using Mathematica [6] for different values of the parameter a . The resulting scale factors and the corresponding magnitude-redshift relations are shown in Figures 4 and 5, respectively.

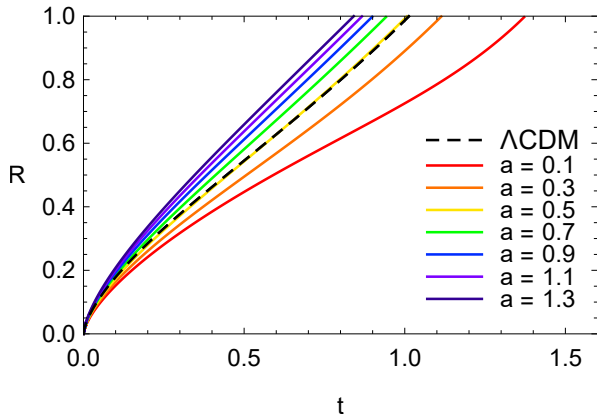


FIG. 4: The time evolution of the scale factor in the string model, for different values of the parameter a , compared to the time evolution in the Λ CDM model. The present time corresponds to $R = 1$. All the curves have the same slope at $R = 1$ so all the models have the same value of the Hubble parameter at the present time.

3. Values of M_U and R_U in the string model

For $a = 0.5$, the solution of equation (A4) is $\eta_0 = 3.6$. Since $H_0 R_U/c = R\eta$, η_0 gives the value of HR_U/c at the present time. The corresponding value of R_U is 15 Gpc, or 4.6×10^{26} m. Neglecting the contribution from radiation, equations (11) and (13) give the relation

$$\frac{GM_U}{R_U c^2} = \frac{\Omega_b}{2} \left(\frac{HR_U}{c} \right)^2. \quad (\text{A5})$$

For $\Omega_b = 0.047$, equation (A5) gives $GM_U/R_U c^2 = 0.30$, corresponding to a value of M_U of 9.3×10^{22} Solar masses.

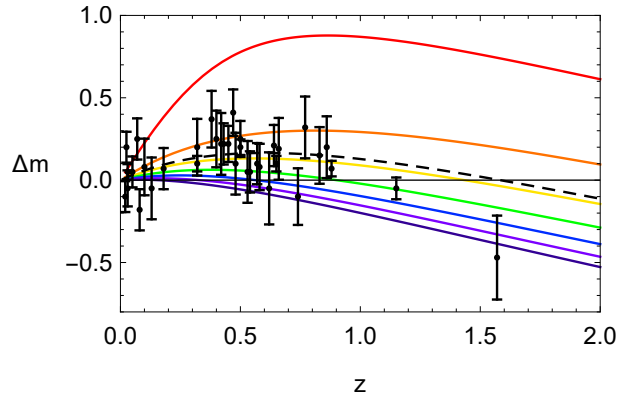


FIG. 5: Plots of residual magnitude against redshift for the string model, with same values of a as in Figure 4, and the the Λ CDM model. The data points are the same as in Figure 2.

The values of HR_U/c and $GM_U/R_U c^2$ are used in the accompanying paper on dark matter [7].

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